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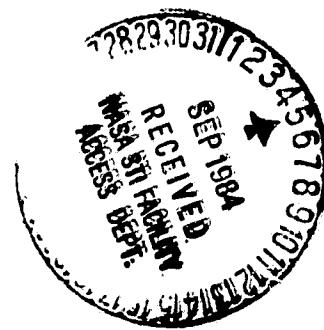
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Double Arch Mirror Study  
Part 2, Engineering Analysis Report

Bijan Iraninejad  
Daniel Vukobratovich  
University of Arizona

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**Double Arch Mirror Study**  
**Part 2, Engineering Analysis Report**

Bijan Iraninejad  
Daniel Vukobratovich  
University of Arizona  
Optical Sciences Center

Prepared for  
Ames Research Center  
under Grant NAG2-220



National Aeronautics and  
Space Administration

**Ames Research Center**  
Moffett Field, California 94035

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## **1. INTRODUCTION**

The mount for the NASA Ames 20-in. double arch mirror must be designed to comply with the following requirements:

- (1) The mirror will be assembled at room temperature (68°F) and used at cryogenic temperatures (-423°F). Therefore, the mount must provide a transition between the fused silica mirror and the aluminum base plate, without reducing the figure quality.
- (2) The mirror will be cooled down and tested in a 1-G gravity field. Therefore, the mount must be designed not to reach its microyield stress in ground testing. The same criterion must be satisfied when the mirror is subjected to the launch load.
- (3) The mount should be designed to survive emergency landing. In this case the survival is defined as developing stresses less than the yield stress. However, an attempt will be made to limit the emergency landing stresses to the microyield level.
- (4) Realistic tolerances should be set for the manufacturing of the mount. The tightness of these tolerances will affect the figure quality. Furthermore, in light of the uncertainty about the magnitude and distribution of the internal stresses in the base plate, the effect of cryogenic temperatures on flatness of the plate cannot be predetermined. However, an acceptable design should allow for the manufacturing tolerances and the existence of tilt in the base plate and should minimize the sensitivity of figure quality to such factors.

- (5) The mount should be designed to maintain the optical alignment of the mirror.

This study is subject to the following assumptions:

- (1) The mirror is assumed to be very stiff. Generally speaking, this is a conservative assumption. A mirror with finite stiffness will result in lower stress in the flexures.
- (2) The mirror material is Corning Code 7740 fused silica. The flexures are assumed to be made of titanium, 6Al-4V ELI, and the base plate as well as the dewar plate are assumed to be made of 6061 aluminum tooling plate. The assumed properties of these materials are given in Table 1.

Table 1. Material Properties

	<u>Corning Code 7740</u>	<u>Titanium 6Al 4V ELI</u>	<u>6061 Aluminum Tooling Plate</u>
Yield stress (psi)	--	240,000	42,000
Microyield stress (psi)	--	115,000	18,000
Modulus of elasticity (psi)	$10 \times 10^6$	$18 \times 10^6$	$10.9 \times 10^6$
Poison ratio	0.17	0.33	0.33
Thermal contraction			
$\frac{L_T - L_{68}}{L_{68}}$	0	$-175 \times 10^{-5}$	$-420 \times 10^{-5}$

- (3) Since the displacements in the mirror are small, it can be assumed that the induced moments and forces are decoupled. Therefore, deflection due to each component of a specific loading can be calculated independently and then, by using the principle of superposition, the deflections can be directly added to find the total deflection of the mirror.
- (4) The range of deflections and stresses in the mirror and the base plate is well within the range of the elastic response. Therefore, once the figure quality is calculated for a specific loading, it can be multiplied by the proper scaling factors to give the figure quality for various magnitudes of the same loading case. Furthermore, it is assumed that the Optical Sciences Center can specify the dimensions and the support mechanism for the base plate. Of course, the base plate design will be compatible with the dewar plate already at Ames.

In this report the forces and/or moments transferred from the mount to the mirror due to cool down, gravity loading, tilt in the base plate, and flexure error as well as magnitude of stress in the flexures under various loadings are calculated analytically. By using the analytical results and utilizing the finite element techniques, the quality of the mirror figure under different conditions is established. By using the magnitude of stress in the flexures and the figure quality in the mirror as the basic criteria, a range of acceptable design dimensions for the mount is found and a specific design is designated. Furthermore, an estimate of the magnitude and distribution pattern of stresses in the mirror is also provided.

## 2. ANALYSIS

### 2.1. Force/Moment Transfer from Mount to Mirror

The proposed mount is shown in Fig. 1, and the flexures are shown in detail in Fig. 2.

#### 2.1.1. Cool Down, Zero Gravity

The mirror has a zero coefficient of thermal expansion. Therefore, cool down in a 0-G environment will induce a radial displacement of the flexure base with respect to the mirror (Fig. 3). For all practical purposes, the flexure top and bottom plates can be assumed to be very stiff (i.e.,  $T = \infty$ ). This assumption is equivalent to analyzing the flexure system as a frame with very stiff girders that in turn degenerates to beam analysis. As shown in Table 2, the above assumption is a conservative one.

Table 2. Comparison between Frame and Beam Analysis

T (in.)	t (in.)	x (in.)	b (in.)	L (in.)	$\Delta$ (in.)	$F_c$ (lb)
$\infty$	0.05	2	2	2.5	0.02652	7.638
0.25	0.05	2	2	2.5	0.02652	7.623

The radial force transferred to the mirror from each flexure blade,  $F_c$ , can be calculated as

$$F_c = E \frac{bt^3}{\ell^3} \Delta \quad (1)$$

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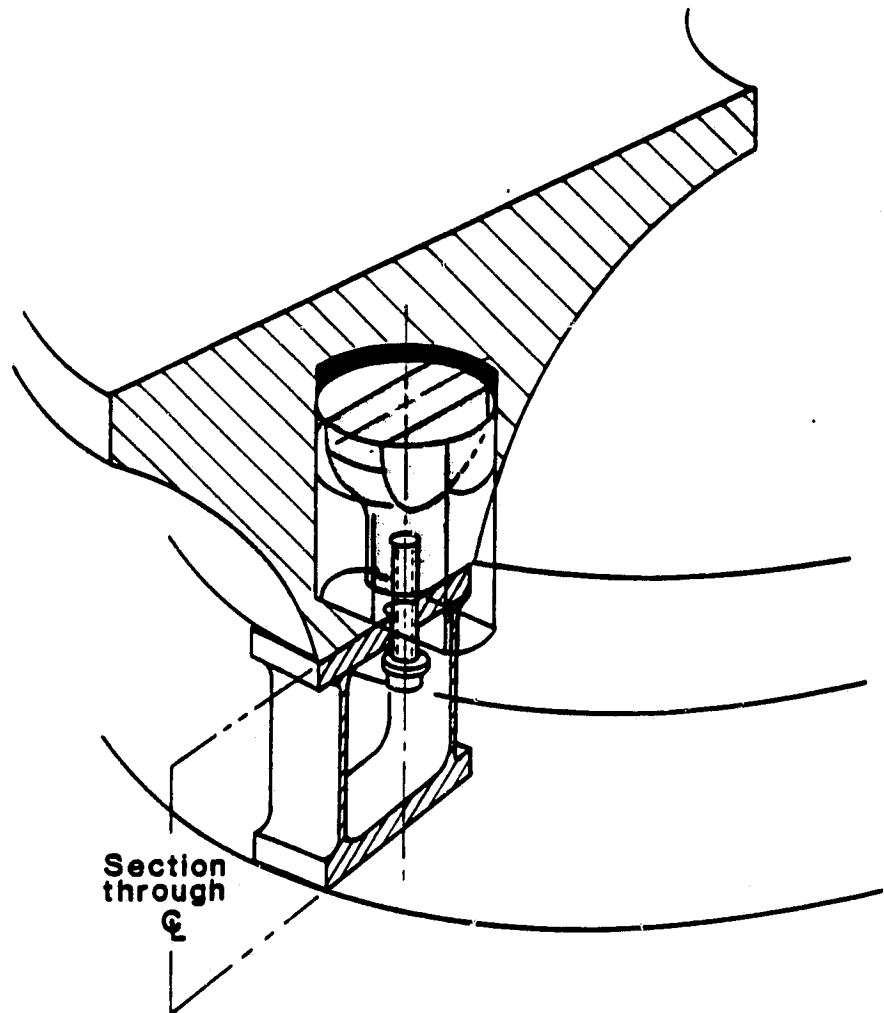


Figure 1. Isometric view of double arch mirror mount.

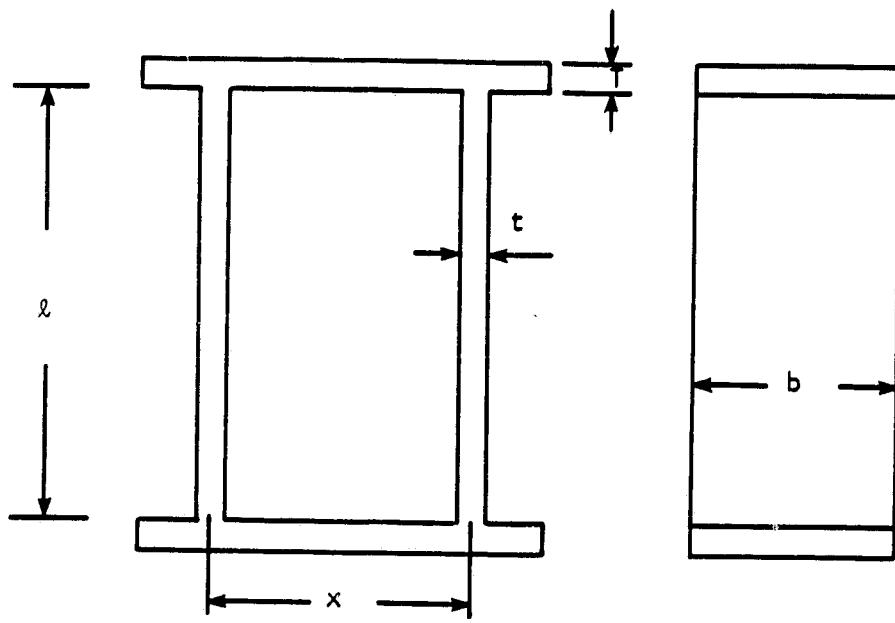


Figure 2. Flexure geometry.

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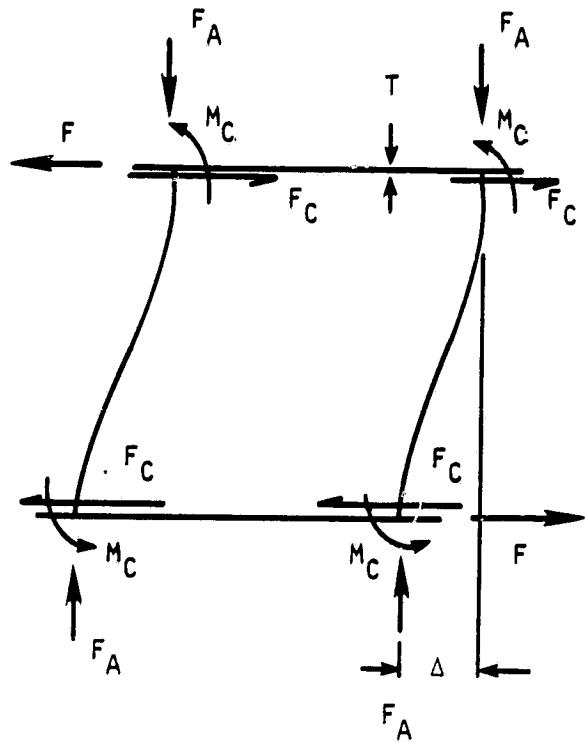


Figure 3. Flexure geometry at cool down.

where

$E$  = modulus of elasticity

$b$  = width of flexure

$t$  = thickness of flexure blade

$\lambda$  = flexure height

$\Delta$  = radial displacement due to cool down.

The total force transferred to the mirror from each flexure,  $F$ , is

$$F = 2F_c.$$

#### 2.1.2. Cool Down, 1-G Loading

The mirror will be tested in cryogenic temperatures under a 1-G gravitational field. The gravity induces axial force to the flexures. In a face-up testing, when the flexures are in compression, the force transferred to the mirror,  $F_{cc}$ , is reduced to

$$F_{cc} = \frac{F_A}{\lambda(\tan\lambda/\lambda - 1)} \Delta, \quad (2)$$

where

$F_A$  = axial force per flexure blade =  $GW/6$

$G$  = gravity loading

$W$  = weight of mirror = 40 lb

$$\lambda = \frac{n\ell}{2}$$

$$n = (F_A/EI)^{1/2} = (12F_A/Ebt^3)^{1/2}.$$

In a face-down testing, where a tensile axial force is generated, the transferred force,  $F_{ct}$ , increases to

$$F_{ct} = \frac{F_A}{l(1 - \tanh \lambda / \lambda)} \Delta. \quad (3)$$

#### 2.1.3. Temperature Effects

Cryogenic temperatures might induce radial tilt ( $\theta_r$ ) and tangential tilt ( $\theta_t$ ) in the base plate. In calculating the effect of such tilts on the mirror, the flexure/base plate interaction plays an important role.

##### (a) Flexure/Base Plate Interaction

Tilt in an infinitely stiff base plate will be totally transferred to an infinitely flexible flexure. On the other hand, if a stiff flexure is mounted on a flexible base plate, no tilt will be transferred to the flexure. Furthermore, a tilt in a stiff base plate will induce negligible moment in a flexible flexure, and as the flexure becomes stiffer, the moment increases.

Both the flexures and base plate have finite stiffness. Therefore, a local tilt in the base plate will be partially transferred to the flexure and partially compensated for in the plate (Fig. 4). This effect can be written as

$$\theta = \theta_f + \theta_p$$

where

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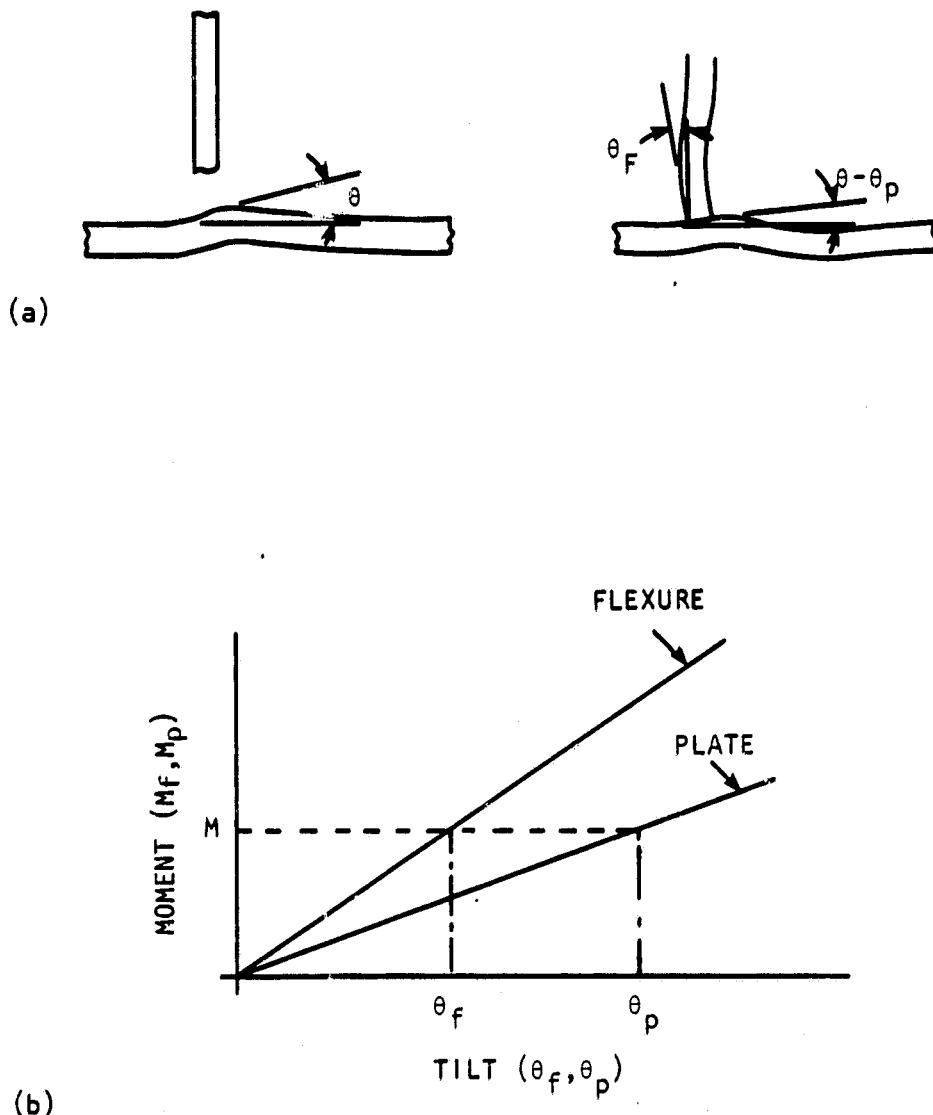


Figure 4. Flexure/base plate interaction

$\theta$  = local tilt in plate

$\theta_f$  = tilt in flexure

$\theta_p$  = tilt in plate.

The equilibrium condition requires that the moment in plate,  $M_p$ , be equal to that of flexure,  $M_f$ , at the plate-flexure interface

$$M_p = M_f = M.$$

All the stresses are limited to the elastic range of the materials. Therefore, tilt and moment in both flexure and plate can be related by linear functions (Fig. 4)

$$\theta_f = f_f M$$

$$\theta_p = f_p M$$

where

$f_f$  = rotational flexibility of flexure

$f_p$  = rotational flexibility of plate.

Therefore, it follows that

$$\theta = (f_f + f_p)M. \quad (4)$$

Any tilt angle calculated for an infinitely stiff plate should be reduced by a factor, R:

$$R = \frac{\theta_f(\text{finite plate stiffness})}{\theta_f(\text{infinitely stiff plate})} = \frac{f_f}{f_f + f_p}. \quad (5)$$

Equation (5) clearly shows the importance of increasing the flexibility of the base plate,  $f_p$ , which is a function of plate size, material, and support system. In this design, a 16-in. diameter, 1/4-in. thick aluminum (tooling) plate with three supports at  $120^\circ$  and on a 7-in. radius is used to maximize the flexibility of the plate. The flexures are mounted to the base plate with a  $60^\circ$  phase angle with respect to the plate supports. For the above

$$f_{pr} = 0.3947 \times 10^{-4} \text{ in.}^{-1} \text{lb}^{-1}$$

$$f_{pt} = 0.2058 \times 10^{-4} \text{ in.}^{-1} \text{lb}^{-1},$$

where  $f_{pr}$  and  $f_{pt}$  are the plate flexibility,  $f_p$ , in radial and tangential directions. Plate flexibility was calculated by finite element techniques.

(b) Radial Tilt

The moment,  $M_r$ , transferred to the mirror due to a radial tilt,  $\theta_r$ , in the base plate is given by

$$M_r = E \frac{bt x^2}{\lambda} R_r \theta_r, \quad (6)$$

where

$x$  = flexure blade separation

$$R_r = \frac{f_{fr}}{f_{fr} + f_{pr}} \quad (7)$$

$$f_{fr} = \frac{\lambda}{2Ebt x^2}. \quad (8)$$

The transferred force,  $f_r$ , will be

$$F_r = \frac{3}{\lambda} M_r. \quad (9)$$

(c) Tangential Tilt

The moment,  $M_t$ , transferred to the mirror due to a tangential tilt,  $\theta_t$ , in the base plate is given by

$$M_t = \frac{E}{3} \frac{tb^3}{\lambda} R_r \theta_t, \quad (10)$$

where

$$R_T = \frac{f_{ft}}{f_{ft} + f_{pt}} \quad (11)$$

$$f_{ft} = \frac{3\lambda}{2Et^3}. \quad (12)$$

The transferred force,  $f_t$ , will be

$$F_t = \frac{3}{\lambda} M_t. \quad (13)$$

(d) Flexure Error

Nonparallelism of the flexures is a possible source of error that might result in transfer of some additional force to the mirror due to

cool down. For a given error,  $\epsilon$ , the induced force,  $F_\epsilon$ , is

$$F_\epsilon = 3E \frac{bt\chi}{\lambda} \left(1 - \frac{\Delta^2}{\lambda^2}\right)^{1/2} \left[1 - \left(1 - \frac{2\Delta\epsilon}{(\lambda^2 - \Delta^2)}\right)\right]^{1/2}, \quad (14)$$

where  $\epsilon$  is the nonparallelism error.

## 2.2. Stress in Flexures

### 2.2.1. Stress in Cryogenic Temperatures

The critical stress develops when the flexures are loaded axially in a cryogenic environment. By using equilibrium conditions, the maximum moment in the flexures,  $M_{ct}$ , can be calculated by (Fig. 3)

$$M_{ct} = \frac{1}{2} (f_{ct}\lambda + F_A\lambda), \quad (15)$$

where  $F_A$  and  $F_{ct}$  are defined by Eqs. (2) and (3).

Therefore, the maximum normal stress,  $\sigma_{ct}$ , and shear stress,  $\tau_{ct}$ , are given by

$$\sigma_{ct} = \frac{6M_{ct}}{bt^2} + \frac{F_A}{bt} \quad (16)$$

$$\tau_{ct} = 1.5 \frac{F_{ct}}{bt}. \quad (17)$$

In the absence of axial loading (0-G, cool down) Eqs. (16) and (17) can be simplified to

$$\sigma_c = 3E \frac{t}{\ell^2} \Delta \quad (18)$$

$$\tau_c = 1.5 \frac{F_c}{bt}. \quad (19)$$

The maximum normal stress occurs at the inner corner of the flexure blades above the base plate whereas the maximum shearing stress is at the physical center of the cross section through the blade. Stress concentration factors will be included in the final calculation.

#### 2.2.2. Side Loading

During launch, emergency landing, or on edge testing, the mirror will be subjected to loading perpendicular to its optical axis. In the worst case, when the side loading is applied in the compliance direction of one of the flexures, the force,  $F_s$ , taken by each blade of the other two flexures can be calculated by

$$F_s = \frac{GW}{2\sqrt{3}}. \quad (20)$$

The maximum normal stress,  $\sigma_s$ , and shearing stress,  $\tau_s$ , are given by

$$\sigma_s = 3F_s \frac{\ell}{tb^2} \quad (21)$$

$$\tau_s = 1.5 \frac{F_s}{bt}. \quad (22)$$

### 2.2.3. Stress due to Mount Error

The existence of local tilt in the base plate will induce stress in the flexures

$$\sigma_r = E \frac{x}{\lambda} R_r \theta_r \quad (23)$$

$$\sigma_t = E \frac{b}{\lambda} R_t \theta_t \quad (24)$$

$$\tau_r = 0.75 \frac{F_r}{bt} \quad (25)$$

$$\tau_t = 0.75 \frac{F_t}{bt}, \quad (26)$$

where

$\sigma_r$  = maximum normal stress due to radial tilt

$\sigma_t$  = maximum normal stress due to tangential tilt

$\tau_r$  = maximum shear stress due to radial tilt

$\tau_t$  = maximum shear stress due to tangential tilt.

Nonparallelism error in the flexures causes additional stress

$$\sigma_e = 3 \frac{F_e \lambda}{bt^2} \quad (27)$$

$$\tau_e = 1.5 \frac{F\epsilon}{bt^2}, \quad (28)$$

where  $\sigma_e$  and  $\tau_e$  are the maximum normal and shearing stress due to flexure error.

#### 2.2.4. Critical Normal Stress

In the worst case, where the stress components are all additive, the maximum normal stress is given by

$$\sigma_{max} = K(\sigma_c + \sigma_s + \sigma_r + \sigma_t + \sigma_f), \quad (29)$$

where  $K$  is the stress concentration factor. As a conservative estimate  $K$  will be assumed to be 1.5.

The maximum shearing stress,  $\tau$ , can be calculated by (Fig. 5)

$$\tau_{max} = (\tau_1^2 + \tau_2^2)^{1/2} \quad (30)$$

where

$$\tau_1 = \tau_t + \tau_s$$

$$\tau_2 = \tau_{ct} + \tau_r + \tau_e.$$

#### 2.3. Influence of Mount on Figure Quality

The principles of linearity and superposition are used in conjunction with finite element techniques to analyze the effect of mount-induced forces/momenta on the mirror.

The finite element model for the double arch mirror consisted of 210 nodes and 168 quadrilateral plate and shell elements (Fig. 6). The

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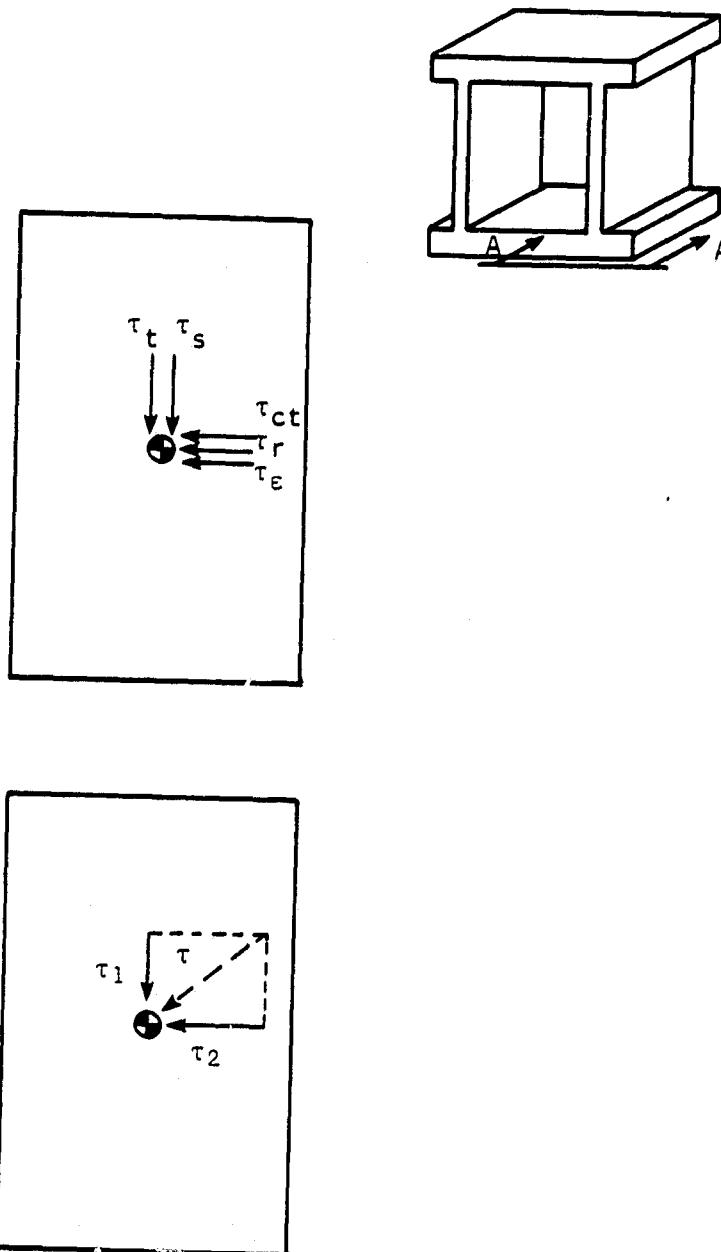


Figure 5. Shearing stress in flexures.

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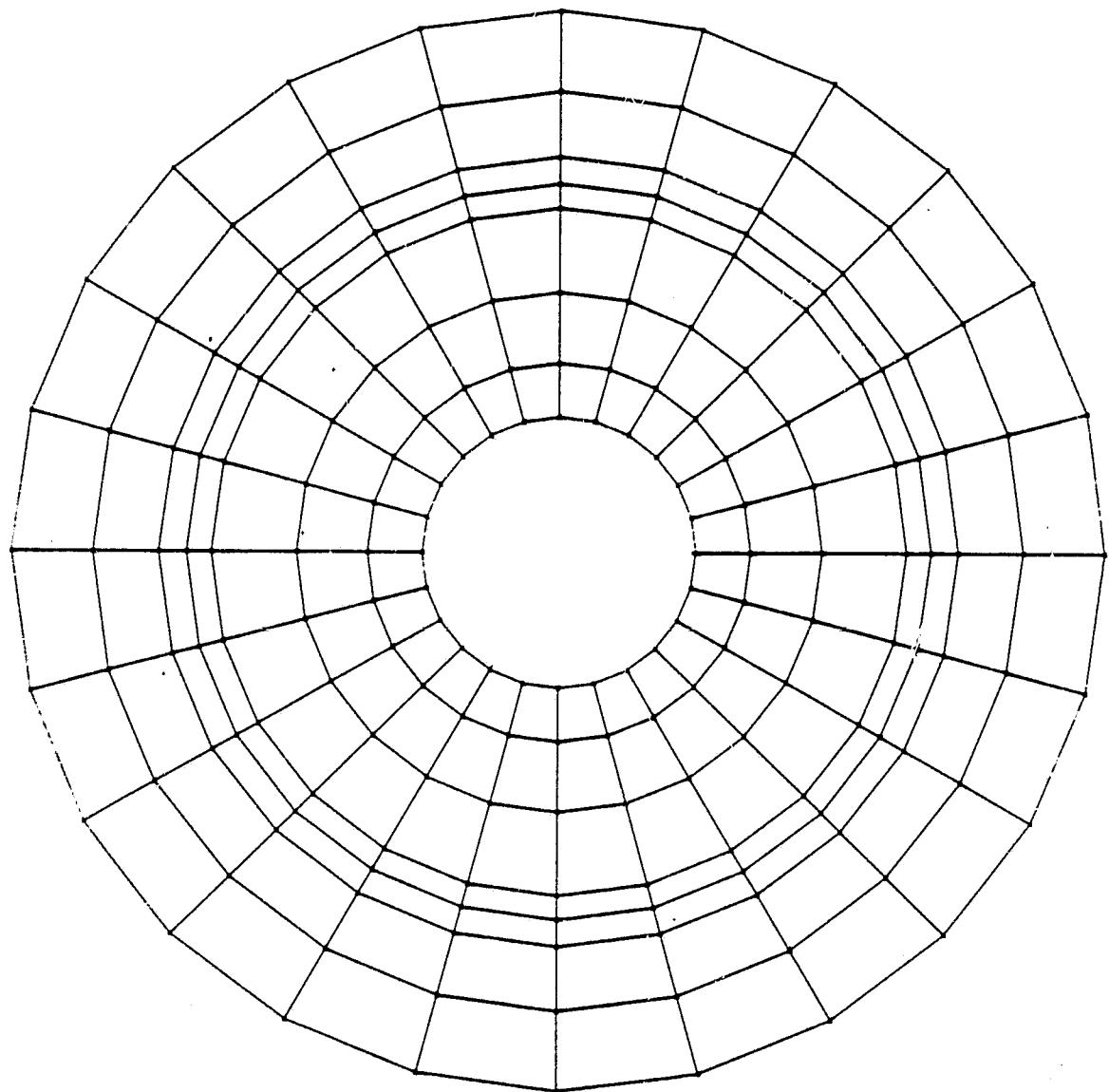


Figure 6. Finite element model of double arch mirror.

software used was SAP IV. This model was used throughout this study to determine the changes in the optical surface due to mount effects. Note that the flexures were implicitly included in the model by directly applying the flexure-induced forces/moment to the mirror base.

Various loadings that the mirror can encounter, either in testing or operation, can be resolved into three different categories (cases 1 through 3). Once the sensitivity of the mirror figure to each loading case is established, then the optical performance of the mirror for any specific flexure design can be determined.

### 2.3.1. Case 1

A set of forces/moment of equal magnitude are applied to the mirror base in the radial direction and  $120^\circ$  apart (Fig. 7a). This case represents the effect of cool down. To arrive at the most fundamental case, we will substitute F and M with  $F_o$  and  $M_o$

$$F_o = F$$

$$M_o = M + Fd$$

where d is the distance between the mirror base and the center of gravity. Note that the line of action of force  $f_o$  passes through the center of gravity of the mirror. Obviously, analyzing the mirror performance with respect to  $M_o$  and  $F_o$  will correspond to a more basic case than M and F.

Figure 8 shows the contour map of the mirror surface when  $M_o = 1$  in.-lb. Also shown in Fig. 8 are the Zernike coefficients and the RMS

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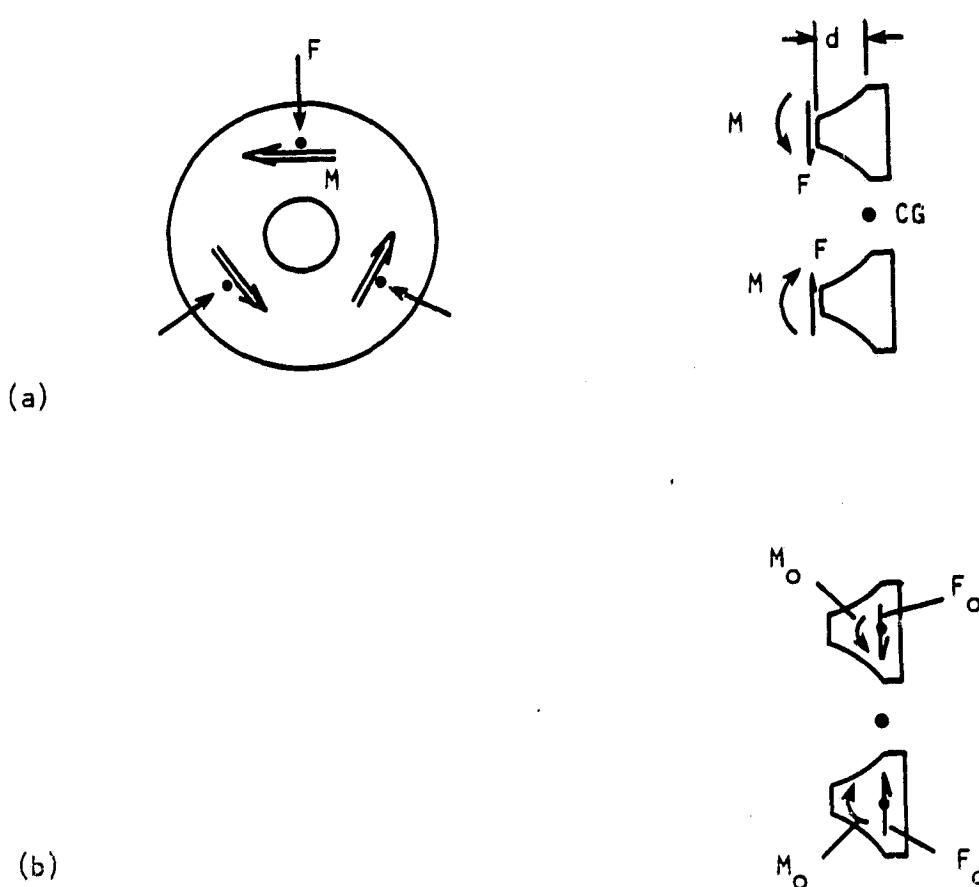


Figure 7. Resolution of forces/momenta.

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## ZERNIKE POLYNOMIAL COEFFICIENTS

## RESIDUAL WAVEFRONT VARIATIONS OVER UNIFORM MESH

PTS	9MS	MAX	HST	SPAN	VOLUM
666.	3.291	11.193	65.845	16.238	14.888

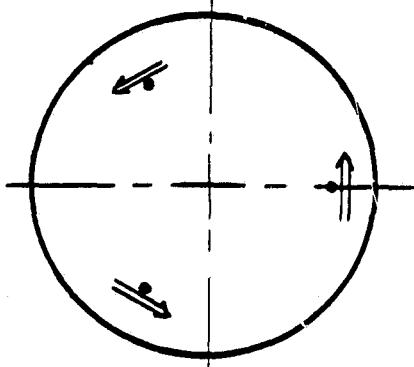


Figure 8. Sensitivity factors for cool down (units of inch x  $10^8$ ).

A schematic of the forces/moment is shown at the lower right of each contour map. The right-hand rule is used to represent moments.

(RMS<sub>co</sub>) value. The values given in Fig. 8 can also serve as the sensitivity factors for the mirror, i.e., each value has units of (in.  $\times$  10<sup>-8</sup>)/(in.-lb). Therefore, for a given moment of  $M = M_m$  ( $M_o = M_m$  and  $F = F_o = 0$ ) the RMS as well as the Zernike coefficients,  $R_n^m$ , can be calculated by

$$\text{RMS} = M_m \times \text{RMS}_{co}$$

$$R_n^m = M_m \times (R_n^m)_{co}$$

where

$\text{RMS}_{co}$  = RMS value when a unit moment is applied.

$(R_n^m)_{co}$  is the Zernike coefficient of mth angular and nth tangential order when a unit moment is applied. For the explicit form of Zernike coefficients see Table 3.

Based on past experience with double arch mirrors, any force that goes through the center of gravity of the mirror (i.e., its line of action is on the plane parallel to the mirror base and passes through the center of gravity) will have a small effect on the overall surface quality. Figures 9 and 10 can be used to illustrate the above. In both figures, a unit radial load is acting on the base. However, Fig. 9 includes the radial force through the center of gravity (i.e.,  $F = 1$ ,  $M = 0$ ,  $F_o = 1$ ,  $M_o = 1 \times d = 1 \times 1.7513 = 1.7513$ ) where as in Fig. 10 only the moment is considered (i.e.,  $M_o = 1.7513$ ,  $F_o = 0$ ). As it is shown, the RMS values differ only by a small factor ( $\Delta(\text{RMS})/\text{RMS} < 0.01$ ). In all the subsequent analyses the effect of any force through the center of gravity has been assumed to be negligible (i.e.,  $F_o = 0$ ).

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Table 3. Zernike Polynomials Used by Fringe in the Order Stored in the Computer

	<u>Polynomial</u>
$R_1^1$	$r \cos \theta$
$R_1^{-1}$	$r \sin \theta$
$R_2^0$	$2r^2 - 1$
$R_2^2$	$r^2 \cos 2\theta$
$R_2^{-2}$	$r^2 \sin 2\theta$
$R_3^1$	$(3r^2 - 2)r \cos \theta$
$R_3^{-1}$	$(3r^2 - 2)r \sin \theta$
$R_4^0$	$6r^4 - 6r^2 + 1$
$R_3^3$	$r^3 \cos 3\theta$
$R_3^{-3}$	$r^3 \sin 3\theta$
$R_4^2$	$(4r^2 - 3)r^2 \cos 2\theta$
$R_4^{-2}$	$(4r^2 - 3)r^2 \sin 2\theta$
$R_5^1$	$(10r^4 - 12r^2 + 3)r \cos \theta$
$R_5^{-1}$	$(10r^4 - 12r^2 + 3)r \sin \theta$
$R_6^0$	$20r^6 - 30r^4 + 12r^2 - 1$
$R_4^4$	$r^4 \cos 4\theta$
$R_4^{-4}$	$r^4 \sin 4\theta$
$R_5^3$	$(5r^2 - 4)r^3 \cos 3\theta$
$R_5^{-3}$	$(5r^2 - 4)r^3 \sin 3\theta$
$R_6^2$	$(15r^4 - 20r^2 + 6)r^2 \cos 2\theta$
$R_6^{-2}$	$(15r^4 - 20r^2 + 6)r^2 \sin 2\theta$
$R_7^1$	$(35r^6 - 60r^4 + 30r^2 - 4)r \cos \theta$
$R_7^{-1}$	$(35r^6 - 60r^4 + 30r^2 - 4)r \sin \theta$
$R_8^0$	$70r^8 - 140r^6 + 90r^4 - 20r^2 + 1$
$R_5^5$	$r^5 \cos 5\theta$
$R_5^{-5}$	$r^5 \sin 5\theta$
$R_6^4$	$(6r^2 - 5)r^4 \cos 4\theta$
$R_6^{-4}$	$(6r^2 - 5)r^4 \sin 4\theta$
$R_7^3$	$(21r^4 - 30r^2 + 10)r^3 \cos 3\theta$
$R_7^{-3}$	$(21r^4 - 30r^2 + 10)r^3 \sin 3\theta$
$R_8^2$	$(56r^6 - 105r^4 + 60r^2 - 10)r^2 \cos 2\theta$
$R_8^{-2}$	$(56r^6 - 105r^4 + 60r^2 - 10)r^2 \sin 2\theta$
$R_9^1$	$(126r^8 - 280r^6 + 210r^4 - 60r^2 + 5)r \cos \theta$
$R_9^{-1}$	$(126r^8 - 280r^6 + 210r^4 - 60r^2 + 5)r \sin \theta$
$R_{10}^0$	$252 r^{10} - 630r^8 + 560r^6 - 210r^4 + 30r^2 - 1$
$R_{12}^0$	$924 r^{12} - 2772r^{10} + 3150r^8 - 1680r^6 + 420r^4 - 42r^2 + 1$

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## ZERNIKE POLYNOMIAL COEFFICIENTS

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## RESIDUAL WAVEFRONT VARIATIONS OVER UNIFORM MESH

RTS	RMS	MAX	MIN	SPAN	VOLUME
664,	5.719	19.475	18.719	28.194	25.73

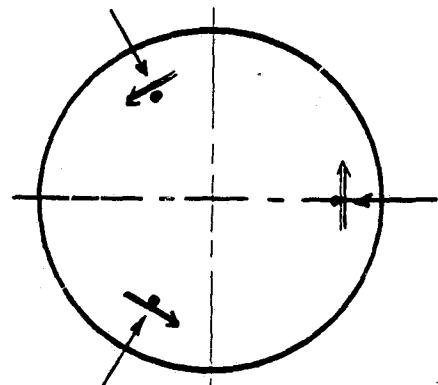


Figure 9. Application of a unit force at the mirror base (units of inch x  $10^8$ ).

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## RESIDUAL WAVEFRONT VARIATIONS OVER UNIFORM MESH

PTS	RMS	MAX	MIN	SPAN	VOLUME
664.	5.744	19.692	16.833	28.437	26.973

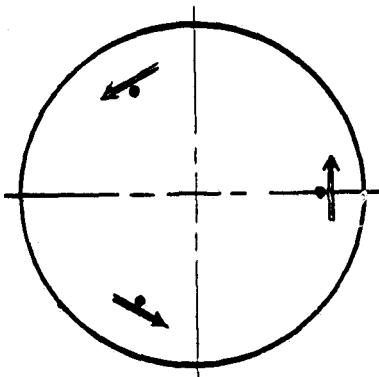


Figure 10. Application of a unit force to the mirror base. The force through the center of gravity is ignored (units of inch  $\times 10^8$ ).

### 2.3.2. Case II

Application of a radial force/moment at only one radial location. This case corresponds to the unpredictable mount error in the radial direction. The contour map, Zernike coefficients, and RMS ( $RMS_{ro}$ ) value are shown in Fig. 11.

### 2.3.3. Case III

Application of a tangential force/moment at only one radial location. This case corresponds to the unpredictable mount error in the radial direction. The contour map, Zernike coefficients, and the RMS ( $RMS_{to}$ ) value are shown in Fig. 12.

By using the three loading cases and the equations given in section 2.3.1, the following equations can be written:

$$RMS_c = (dF)(RMS_{co}) \text{ for cool-down effect} \quad (31)$$

$$RMS_r = (M_r + dF_r)(RMS_{ro}) \text{ for radial tilt} \quad (32)$$

$$RMS_t = (M_t + dF_t)(RMS_{to}) \text{ for tangential tilt} \quad (33)$$

$$RMS_f = (2dF_f)(RMS_{ro}) \text{ for flexure error.} \quad (34)$$

Table 4 gives a listing of the RMS sensitivity factors.

Table 4. RMS Sensitivity Factors

	RMS (in. $\times 10^6$ )
$RMS_{co}$	0.0329
$RMS_{ro}$	0.0271
$RMS_{to}$	0.0927

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## ZERNIKE POLYNOMIAL COEFFICIENTS

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-२. ९०८०	-२. ९०८१	-२. ९०८२	-२. ९०८३	-२. ९०८४	-२. ९०८५	-२. ९०८६	-२. ९०८७
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## RESIDUAL WAVEFRONT VARIATIONS OVER UNIFORM MESH

PTS RMS MAX MIN SPAN VOLUME  
664. 2.789 5.889 -4.281 15.886 27.389

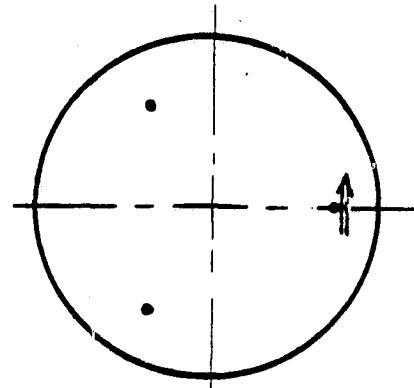


Figure 11. Sensitivity factors for radial tilt (units of inch x  $10^{-8}$ ).

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## ZERNIKE POLYNOMIAL COEFFICIENTS

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-१. ४४९८	.७४८१	-१. ग्रन्त	-०. ३५७१	.८८८८	.३२३७	-०. ८८८८	-०. २४९६
.१६००	.८८८८	-१. ग्रन्त	.८८८८				

## RESIDUAL WAVEFRONT VARIATIONS OVER UNIFORM MESH

PT3 RMS MAX MIN SPAN VOLUME  
664. 9.273 21.397 -21.397 43.613 63.439

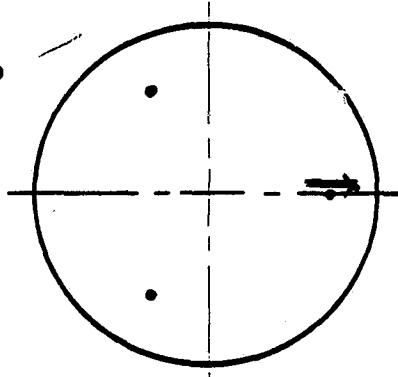


Figure 12. Sensitivity factor for tangential tilt (units of inch x  $10^8$ ).

### 3. FLEXURE DESIGN

#### 3.1. Design Range

The flexure design was directly controlled by several factors.

##### 3.1.1. Manufacturing Requirements

Based on the dimensional limitations imposed on the design by the manufacturing process and also based on past experience with flexures, the design parameters were constrained to a feasible region given by

$$0.03 \text{ in.} < t < 0.04 \text{ in.}$$

$$2.0 \text{ in.} < l < 4.0 \text{ in.}$$

$$0.6 \text{ in.} < x < 2.0 \text{ in.}$$

$$0.4 \text{ in.} < b < 2.0 \text{ in.}$$

(35)

##### 3.1.2. Surface Quality Requirements

Establishing accurate estimates of possible mount error or temperature effects, if possible, will require considerable time and effort. Therefore, instead of imposing performance requirements on the overall surface quality, we will limit the contribution of independent sources of figure deterioration. Furthermore, we will quantify the surface quality by the root mean square (RMS) value of the differences between the deformed mirror surface and the undeformed surface measured at 644 regularly spaced grid points. In doing so, it is assumed that tilt and focus terms can be corrected by the telescope system. The following requirements were imposed on the design:

$$\text{RMS}_c < 1 \times 10^{-6} \text{ in. due to cool down}$$

$$\text{RMS}_s < 2.5 \times 10^{-6} \text{ in. due to radial tilt}$$

$$RMS_t < 2.5 \times 10^{-6} \text{ in. due to tangential tilt} \quad (36)$$

$$RMS_f < 2.5 \times 10^{-6} \text{ in. due to flexure error.}$$

### 3.1.3. Stress Requirements

The maximum stress in the flexures will be designed not to exceed the microyield limit, either in operation or during launch. However, in an emergency, landing stresses as high as the yield point will be allowable. A safety factor of 2 will be used:

$$\sigma_c < 57,500 \text{ psi during O-G cool down}$$

$$\sigma_{ct} < 57,500 \text{ psi during launch}$$

$$\sigma_{ct} < 120,000 \text{ psi during emergency landing} \quad (37)$$

$$\tau < 42,000 \text{ psi during emergency landing.}$$

It would have been ideal to optimize the design for minimum RMS in the mirror and minimum stress in the flexures. However, these two objectives are in direct conflict. An unsuccessful attempt was made to optimize the design for a minimum RMS in the mirror by using the OPTLIB (Ref. 1) optimization package. The failure of this approach might be due to the highly nonlinear nature of the problem. Therefore, in the absence of a better approach, a parametric study of the feasible region defined by Eqs. (35), (36), and (37) was performed. As a result of this study the following design parameters were chosen:

$$t = 0.04 \text{ in.}$$

$$l = 3.60 \text{ in.}$$

$$x = 1.00 \text{ in.}$$

$$b = 0.60 \text{ in.}$$

(38)

An inclusive list of the feasible design region is given in Appendix A.

### 3.2 Design Calculations

In this section we illustrate the application of the analytical approach, as outlined in section 2, by applying it to the proposed design as given by Eq. (38). Numbers on the right-hand side denote the equation used.

#### 3.2.1. 0-G Cool Down

$$\Delta = 0.0286 \text{ in.}$$

$$F_c = 18 \times 10^6 \times \frac{(0.6)(0.04)^3}{(3.6)^3} \times 0.0286 \\ = 0.424 \text{ lb} \quad (1)$$

$$\sigma_c = 3 \times 18 \times 10^6 \times \frac{0.04}{(3.6)^2} \times 0.0286 = 4766.7 \text{ psi} \quad (18)$$

$$\tau_c = 1.5 \times \frac{0.424}{(0.6)(0.04)} = 26.5 \text{ psi} \quad (19)$$

$$RMS_c = (1.7513 \times 2 \times 0.424)(0.0329 \times 10^{-6}) \\ = 0.049 \times 10^{-6} \text{ in.} \quad (31)$$

For a contour map see Fig. 13.

#### 3.2.2. 1-G Cool Down. Face Down Testing

$$F_A = \frac{1 \times 40}{6} = 6.667 \text{ lb} \quad (2)$$

$$n = \left[ \frac{12 \times 6.667}{18 \times 10^6 \times 0.6 \times (0.04)^3} \right]^{1/2} = 0.340 \quad (2)$$

$$\lambda = \frac{0.340 \times 3.6}{2} = 0.612 \quad (2)$$

$$F_{ct} = \frac{6.667}{3.6(1 - 1/0.612 \tanh 0.612)} \times 0.0286 = 0.487 \text{ lb} \quad (3)$$

$$M_{ct} = \frac{1}{2} (0.487 \times 3.6 + 6.667 \times 0.0286) = 0.972 \text{ in.-lb} \quad (15)$$

$$\sigma_{ct} = \frac{6 \times 0.972}{0.6 \times (0.04)^2} + \frac{6.667}{0.6 \times 0.04} = 6354.0 \text{ psi} \quad (16)$$

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.0000	.0025	-.0000	.0000	-.0000	.0000	-.0000	.0000	.0112
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.0000	.0000	-.0038	.0000	.0000	.0000	.0000	-.0000	.0000

RESIDUAL WAVEFRONT VARIATIONS OVER UNIFORM MESH

PTS	RMS	MAX	MIN	SPAN	VOLUME
664.	.249	.166	.075	.241	.221

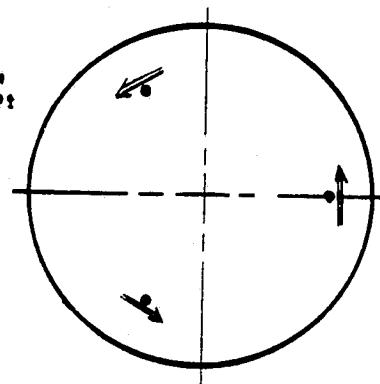


Figure 13. Contour map of double arch mirror surface at cool down with O-G loading (units of inch x 10<sup>6</sup>).

$$\tau_{ct} = 1.5 \times \frac{0.487}{0.6 \times 0.04} = 30.4 \text{ psi} \quad (17)$$

$$\begin{aligned} RMS_{ct} &= (1.7513 \times 2 \times 0.487)(0.0329 \times 10^{-6}) \\ &= 0.056 \times 10^{-6} \end{aligned} \quad (31)$$

For a contour map see Fig. 14.

### 3.2.3. Launch. Flexures in Tension

$$F_A = \frac{3.2 \times 40}{6} = 21.333 \text{ lb} \quad (2)$$

$$\sigma_{ct} = 9837.8 \text{ psi} \quad (16)$$

$$\tau_{ct} = 39.1 \text{ psi} \quad (17)$$

$$F_s = \frac{0.8 \times 40}{2\sqrt{3}} = 9.238 \text{ lb} \quad (20)$$

$$\sigma_s = \frac{3 \times 9.238 \times 3.6}{(0.04)(0.6)^2} = 6928.2 \text{ psi} \quad (21)$$

$$\tau_s = 1.5 \times \frac{9.238}{0.6 \times 0.04} = 577.3 \text{ psi} \quad (22)$$

### 3.2.4. Emergency Landing. Flexures in Tension

$$F_A = \frac{4.5 \times 40}{6} = 30.0 \text{ lb} \quad (2)$$

$$\sigma_{ct} = 11891.3 \text{ psi} \quad (16)$$

$$\tau_{ct} = 44.2 \text{ psi} \quad (17)$$

$$F_s = \frac{4.5 \times 40}{2\sqrt{3}} = 51.962 \text{ lb} \quad (20)$$

$$\sigma_s = 38971.1 \text{ psi} \quad (21)$$

$$\tau_s = 3247.6 \text{ psi} \quad (22)$$

### 3.2.5. Radial Tilt

Assume  $\theta_r = 0.001 \text{ rad}$

$$f_{fr} = \frac{3.6}{2 \times 18 \times 10^6 \times 0.6 \times 0.04 \times (1)^2} = 4.167 \times 10^{-6} \text{ in.}^{-1} \text{lb}^{-1} \quad (8)$$

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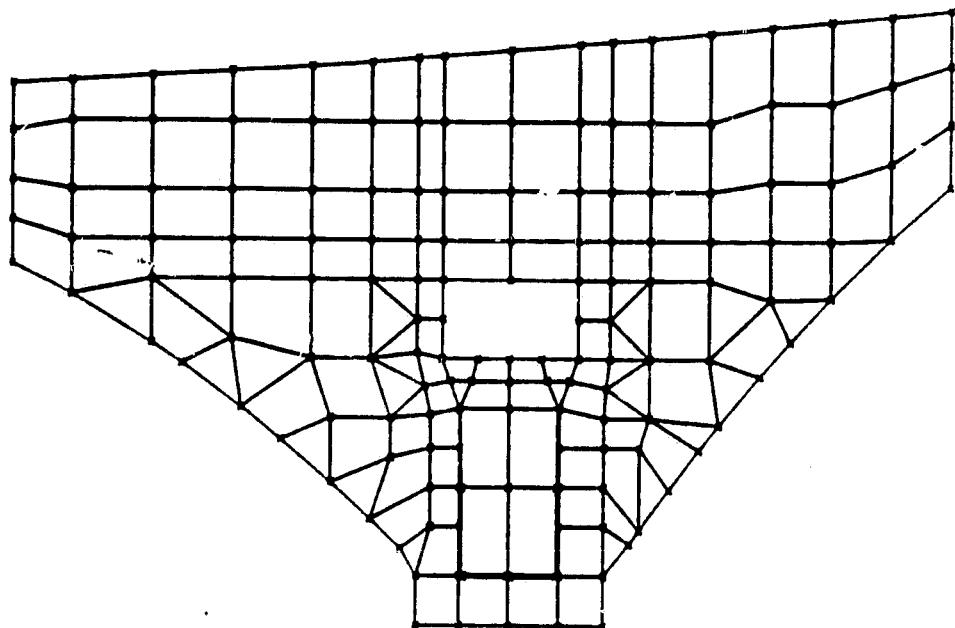


Figure 14. Axisymmetric finite element model of double arch mirror.

$$R_r = \frac{4.167 \times 10^{-6}}{4.167 \times 10^{-6} + 0.3947 \times 10^{-4}} = 0.095 \quad (7)$$

$$M_r = 18 \times 10^6 \times \frac{0.6 \times 0.04 \times (1)^2}{3.6} \times 0.095 \times 0.001 = 11.458 \text{ in.-lb} \quad (16)$$

$$F_r = \frac{3}{3.6} \times 11.458 = 9.549 \text{ lb} \quad (9)$$

$$\sigma_r = 18 \times 10^6 \times \frac{1}{3.6} \times 0.095 \times 0.001 = 477.7 \text{ psi} \quad (23)$$

$$\tau_r = 298.4 \text{ psi} \quad (25)$$

$$\begin{aligned} RMS_r &= (11.458 + 1.7513 \times 9.549)(0.0271 \times 10^{-6}) \\ &= 0.763 \times 10^{-6} \text{ in.} \end{aligned} \quad (32)$$

### 3.2.6. Tangential Tilt

Assume  $\theta_t = 0.001 \text{ rad}$

$$f_{ft} = \frac{3 \times 3.6}{2 \times 18 \times 10^6 \times 0.04 \times (0.6)^3} = 3.472 \times 10^{-5} \text{ in.}^{-1} \text{ lb}^{-1} \quad (12)$$

$$R_t = \frac{3.472 \times 10^{-5}}{3.472 \times 10^{-5} + 0.2058 \times 10^{-4}} = 0.628 \quad (11)$$

$$M_t = \frac{18 \times 10^6}{3} \times \frac{0.04 \times (0.6)^3}{3.6} \times 0.628 \times 0.001 = 9.041 \text{ in.-lb} \quad (10)$$

$$F_t = \frac{3}{3.6} \times 9.041 = 7.534 \text{ lb} \quad (13)$$

$$\sigma_t = 18 \times 10^6 \times \frac{0.6}{3.6} \times 0.628 \times 0.001 = 1883.6 \text{ psi} \quad (24)$$

$$\tau_t = 0.75 \times \frac{7.534}{0.6 \times 0.04} = 235.5 \text{ psi} \quad (26)$$

$$\begin{aligned} RMS_t &= (9.041 + 1.7513 \times 7.534)(0.09275 \times 10^{-6}) \\ &= 2.062 \times 10^{-6} \text{ in.} \end{aligned} \quad (33)$$

### 3.2.7. Flexure Error

Assume  $\epsilon = 0.001 \text{ in.}$

$$F_e = 3 \times 18 \times 10^6 \times \frac{0.6 \times 0.64 \times 1}{3.6} \\ \times \left(1 - \frac{(0.0286)^2}{(3.6)^2}\right)^{1/2} \left[1 - \left(1 - \frac{2 \times 0.0286 \times 0.001}{(3.6)^2 - (0.0286)^2}\right)^{1/2}\right] \\ = 0.794 \text{ lb} \quad (14)$$

$$\sigma_e = 3 \times \frac{0.794 \times 3.6}{0.6 \times (0.04)^2} = 8938.1 \text{ psi} \quad (27)$$

$$\tau_e = 1.5 \times \frac{0.794}{0.6 \times (0.04)^2} = 1241.4 \text{ psi} \quad (28)$$

$$RMS_f = (1.7513 \times 2 \times 0.794)(0.0271 \times 10^{-6}) \\ = 0.075 \times 10^{-6} \text{ in.} \quad (34)$$

### 3.2.8. Presence of Mount Error

By including the stress concentration factor and the above calculation, the critical stresses can be obtained.

(a) O-G cool down in the presence of mount error:

$$\sigma_{\max} = 1.5(4766.7 + 0 + 477.4 + 1883.6 + 8938.1) \\ = 24097 \text{ psi} \quad (29)$$

$$\tau_1 = 235.5 + 0 = 235.5 \text{ psi} \quad (30)$$

$$\tau_2 = 26.5 + 298.4 + 1241.4 = 1566.3 \text{ psi}$$

$$\tau_{\max} = (235.5^2 + 1655.3^2)^{1/2} = 1583.9 \text{ psi}$$

(b) Launch in the presence of mount error:

$$\sigma_{\max} = 1.5(9837.8 + 6928.2 + 477.4 + 1883.6 + 8938.1) \\ = 42098.1 \text{ psi}$$

$$\tau_1 = 235.5 + 577.3 = 812.8 \text{ psi}$$

$$\tau_2 = 39.1 + 298.4 + 1241.4 = 1578.9 \text{ psi}$$

$$\tau_{\max} = (812.8^2 + 1578.9^2)^{1/2} = 1775.8 \text{ psi}$$

(c) Emergency landing in the presence of mount error:

$$\sigma_{\max} = 1.5(11891.3 + 38971.3 + 477.4 + 1883.6 + 8938.1)$$
$$= 93242.5 \text{ psi}$$

$$\tau_1 = 235.5 + 3247.6 = 3483.1 \text{ psi}$$

$$\tau_2 = 44.2 + 298.4 + 1241.4 = 1584.0 \text{ psi}$$

$$\tau_{\max} = (3483.1^2 + 1584.0^2)^{1/2} = 3826.4 \text{ psi.}$$

Note that the RMS values are not directly additive. Appendix B gives examples of various cases where one or more mount errors are present.

#### **4. STRESS IN THE MIRROR**

To estimate the stress distribution inside the mirror due to the clamping force, a study was performed using the finite element method. The finite element model consisted of 136 triangular and quadrilateral axisymmetric elements (Fig. 14). It was assumed that the friction between the mirror base and the flexure top prevents any motion of the two with respect to each other. The stress profiles are shown in Figs. 15 through 20. The stress components are defined in Fig. 21.

For a clamping force of 40 lb, the maximum compressive stress is 501 psi and the maximum tensile stress is 235 psi, both of which are within the acceptable range. Note that these results can be scaled linearly for various clamping forces within the elastic range of the material.

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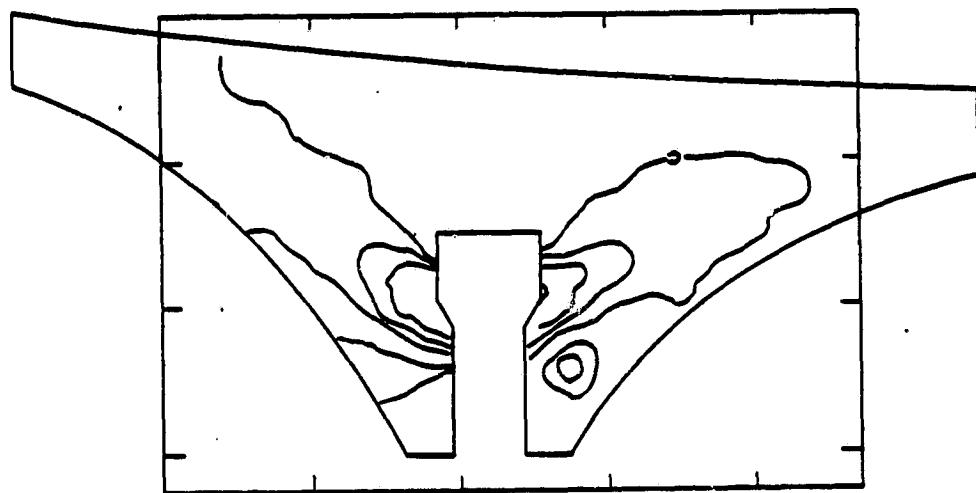


Figure 15. Stress profile of the normal stress,  $S_{11}$ .

Contour interval: 30 psi  
Maximum normal stress: 129  
Minimum normal stress: -81

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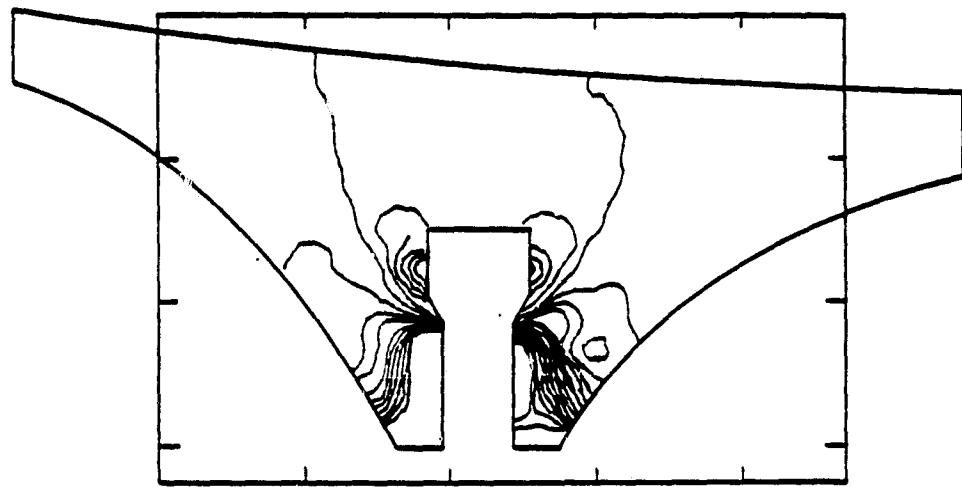


Figure 16. Stress profile of the normal stress,  $S_{22}$ .

Contour interval: 30 psi  
Maximum compressive stress: -483 psi  
Maximum tensile stress: +170 psi

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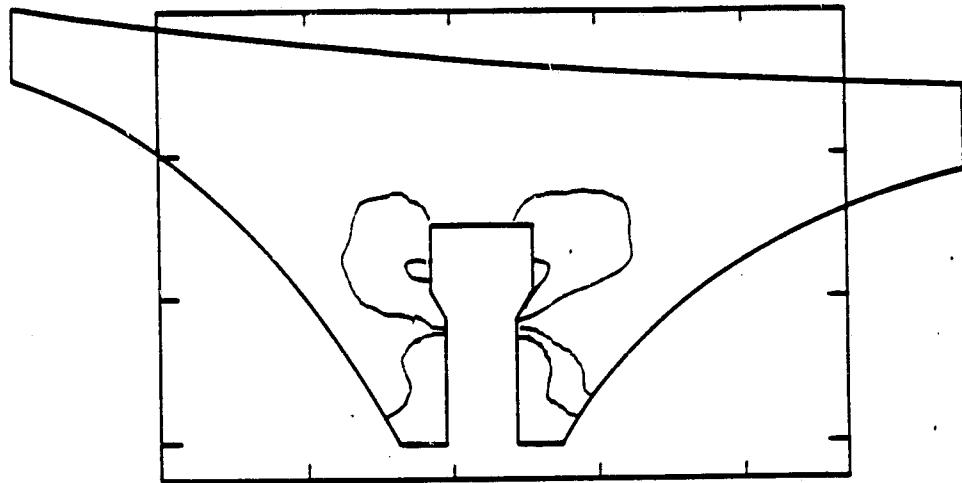


Figure 17. Stress profile of the normal stress,  $S_{33}$ .

Contour interval: 30 psi  
Maximum compressive stress: -88 psi  
Maximum tensile stress: +50 psi

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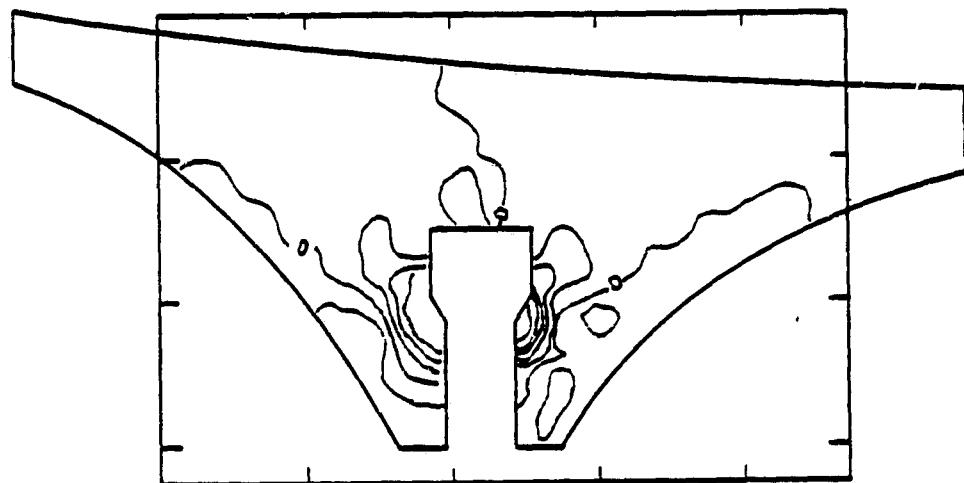


Figure 18. Stress profile for the shearing stress, S12.

Contour intervals: 30 psi

Maximum shearing stress: 210 psi

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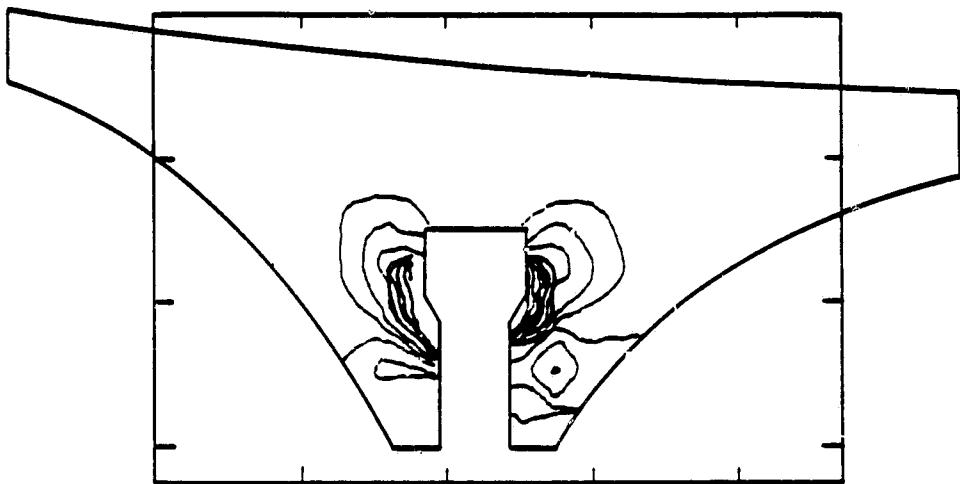


Figure 19. Stress profile of the major principal normal stress,  $S_{\max}$ .

Contour interval: 30 psi

Maximum major principal stress: 236 psi

Minimum major principal stress: -63 psi

STRESS PROFILE  
OF POAR CUBE

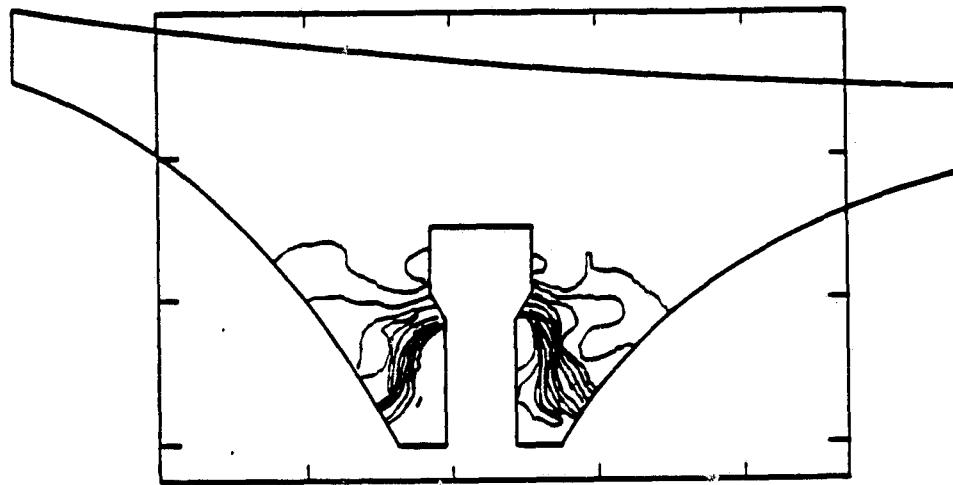


Figure 20. Stress profile of the minor principal normal stress,  $S_{min}$ .

Contour interval: 30 psi  
Maximum minor principal stress: 28 psi  
Minimum minor principal stress: -501 psi

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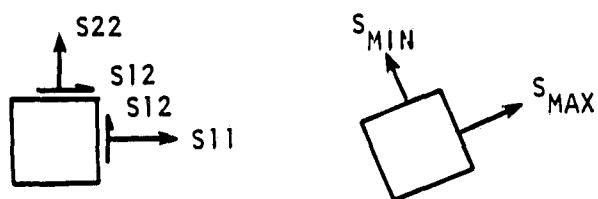
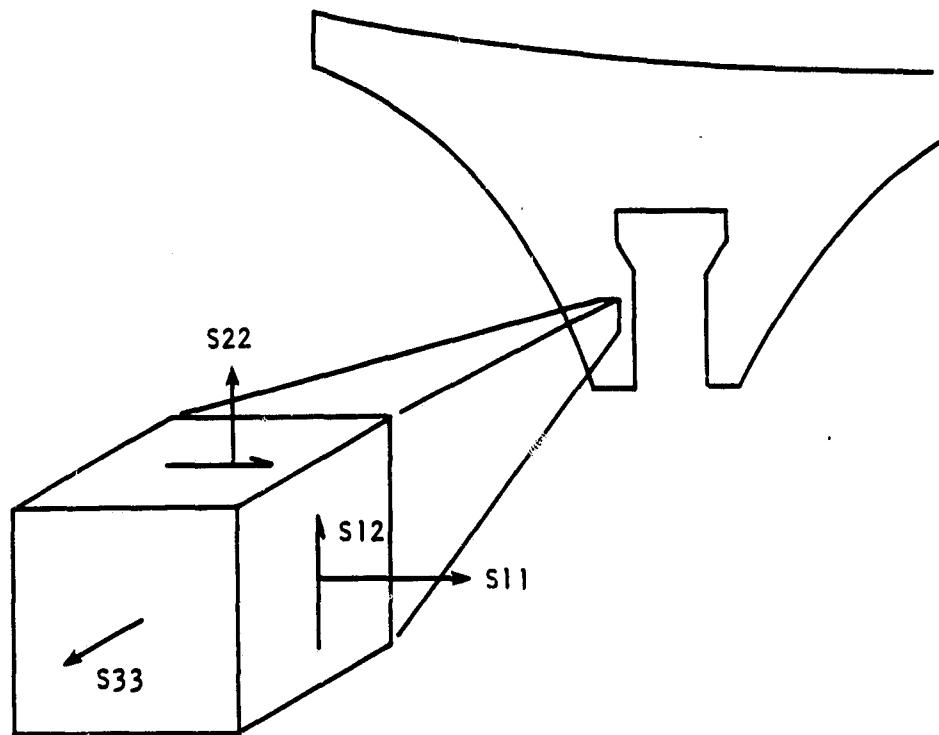


Figure 21. Definition of the stress components in axisymmetric model.

## **5. CONCLUSION**

Figure 22 is a schematic of the proposed flexure design. The maximum normal and shear stress for this design is within the acceptable range. The effect of cool down as well as any mount-induced forces/moment are examined, and the contribution of any single error is limited to  $2.5 \times 10^{-6}$  in. RMS. The maximum compression and tensile stress inside the mirror due to a 40-lb clamping force are 501 psi and 235 psi, respectively.

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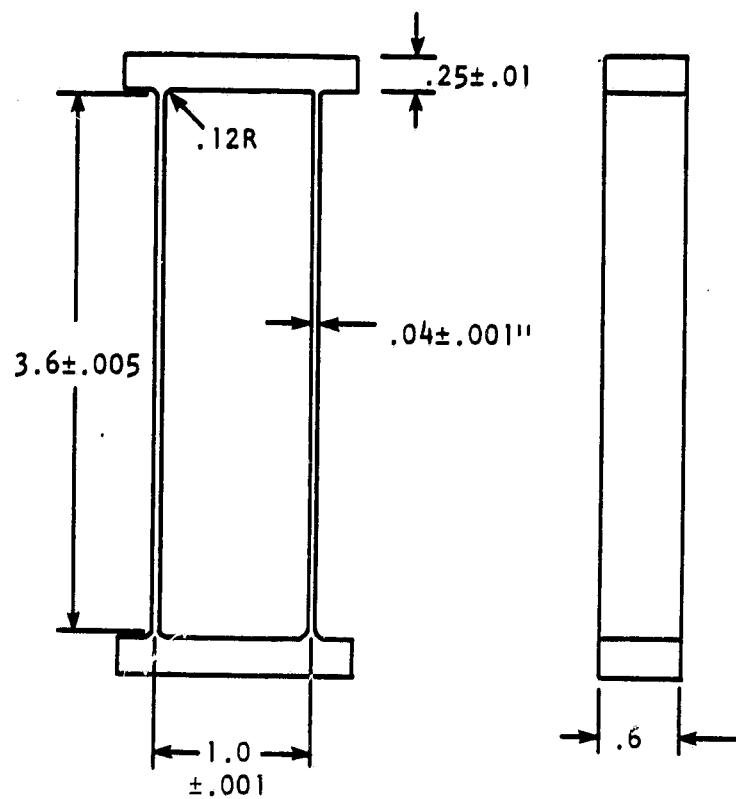


Figure 22. Proposed flexure design.

#### **REFERENCES**

1. G. A. Gabriele and K. M. Ragsdell, "OPTLIB: An Optimization Program Library," Purdue Research Foundation, 1979.

## **APPENDIX A**

In this appendix an inclusive list of feasible design parameters with the calculated stress and RMS values is provided.

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Normal stress (psi)				RMS (inch x 10 <sup>6</sup> )				Shear stress			
0G	cool	Launch plus mount	Emergency landing plus mount	OG cool down	Radial tilt	Tang. tilt	Flex. error	Launch plus mount	Emergency landing plus mount	Emergency landing plus mount	Launch plus mount error
0.336	2.4	5.6	59365	116360	75	75	187	2430	5396	5160	116360
0.336	3.5	6.5	26015	52017	71	71	246	2120	5096	5049	52017
0.336	3.5	6.5	32927	57409	68	68	246	2576	5209	5179	57409
0.336	3.7	6.6	25012	92023	67	67	246	1676	5079	5071	92023
0.336	3.7	6.6	29365	54686	63	63	164	1679	4071	4071	54686
0.336	3.7	6.6	22561	48555	62	62	164	2255	5031	5031	48555
0.336	3.7	6.6	24377	51489	61	61	163	1691	4103	4103	51489
0.336	3.7	6.6	28666	67214	61	61	163	1691	4103	4103	67214
0.336	3.7	6.6	23609	59466	60	60	163	2529	4103	4103	59466
0.336	3.7	6.6	21334	46193	59	59	163	1692	4113	4113	46193
0.336	3.7	6.6	26524	46162	58	58	163	1692	4113	4113	46162
0.336	3.7	6.6	43372	69160	58	58	163	1692	4113	4113	69160
0.336	3.7	6.6	29778	46026	58	58	163	2529	4103	4103	46026
0.336	3.7	6.6	32067	49916	58	58	163	2529	4103	4103	49916
0.336	3.7	6.6	36301	53299	57	57	163	3391	3196	3196	53299
0.336	3.7	6.6	39713	56961	57	57	163	1664	3796	3796	56961
0.336	3.7	6.6	29495	36419	56	56	163	1664	3796	3796	36419
0.336	3.7	6.6	31640	68695	56	56	163	2529	4103	4103	68695
0.336	3.7	6.6	23082	41209	56	56	163	2529	4103	4103	41209
0.336	3.7	6.6	24664	44117	55	55	163	2529	4103	4103	44117
0.336	3.7	6.6	28577	47221	55	55	163	2529	4103	4103	47221
0.336	3.7	6.6	32924	49948	54	54	163	2529	4103	4103	49948
0.336	3.7	6.6	31640	52092	54	54	163	2529	4103	4103	52092
0.336	3.7	6.6	36425	53648	54	54	163	2529	4103	4103	53648
0.336	3.7	6.6	19888	37319	54	54	163	2529	4103	4103	37319
0.336	3.7	6.6	21568	39591	54	54	163	2529	4103	4103	39591
0.336	3.7	6.6	24898	42897	53	53	163	2529	4103	4103	42897
0.336	3.7	6.6	26678	46677	52	52	163	2529	4103	4103	46677
0.336	3.7	6.6	20261	47218	52	52	163	2529	4103	4103	47218
0.336	3.7	6.6	31961	46966	51	51	163	2529	4103	4103	46966
0.336	3.7	6.6	36513	52532	51	51	163	2529	4103	4103	52532
0.336	3.7	6.6	37174	55173	51	51	163	2529	4103	4103	55173
0.336	3.7	6.6	12375	35909	50	50	163	2529	4103	4103	35909
0.336	3.7	6.6	19616	31691	50	50	163	2529	4103	4103	31691
0.336	3.7	6.6	21892	46467	50	50	163	2529	4103	4103	46467
0.336	3.7	6.6	24199	42773	50	50	163	2529	4103	4103	42773
0.336	3.7	6.6	26528	47492	50	50	163	2529	4103	4103	47492
0.336	3.7	6.6	31211	47448	49	49	163	2529	4103	4103	47448
0.336	3.7	6.6	33598	52172	49	49	163	2529	4103	4103	52172
0.336	3.7	6.6	15949	35943	49	49	163	2529	4103	4103	35943
0.336	3.7	6.6	17956	37106	49	49	163	2529	4103	4103	37106
0.336	3.7	6.6	24674	40606	48	48	163	2529	4103	4103	40606
0.336	3.7	6.6	22674	41224	48	48	163	2529	4103	4103	41224
0.336	3.7	6.6	28149	43319	47	47	163	2529	4103	4103	43319
0.336	3.7	6.6	26281	45119	47	47	163	2529	4103	4103	45119
0.336	3.7	6.6	24694	47554	47	47	163	2529	4103	4103	47554
0.336	3.7	6.6	30576	40606	47	47	163	2529	4103	4103	40606
0.336	3.7	6.6	28601	39198	46	46	163	2529	4103	4103	39198
0.336	3.7	6.6	26972	43214	45	45	163	2529	4103	4103	43214
0.336	3.7	6.6	26813	42246	45	45	163	2529	4103	4103	42246
0.336	3.7	6.6	26802	36203	45	45	163	2529	4103	4103	36203
0.336	3.7	6.6	31171	44572	45	45	163	2529	4103	4103	44572
0.336	3.7	6.6	31231	46684	45	45	163	2529	4103	4103	46684
0.336	3.7	6.6	17168	31611	44	44	163	2529	4103	4103	31611
0.336	3.7	6.6	14922	32725	44	44	163	2529	4103	4103	32725
0.336	3.7	6.6	26716	34506	44	44	163	2529	4103	4103	34506

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Normal stress (psi)				Shear stress			
OG	cool	Emergency landing	RMS (inch x 10 <sup>6</sup> )	OG	cool	Emergency landing	Launch plus mount error
b (in.)	x (in.)	plus mount error	plus mount error	OC	cool down	plus mount error	plus mount error
.950	1.2	.60	.22575	36417	.79	.77	.272
.950	1.4	.60	24466	36494	.769	.76	.272
.950	1.6	.60	26311	62623	.769	.76	.272
.950	1.8	.60	26186	42658	.769	.76	.272
.950	2.0	.60	30948	43951	.660	.76	.272
.950	2.2	.60	15719	36043	.768	.653	.272
.950	2.4	.60	17318	31613	.761	.669	.272
.950	2.6	.60	14967	31229	.764	.669	.272
.950	2.8	.60	26596	34913	.764	.713	.272
.950	3.0	.60	22263	16517	.713	.716	.272
.950	3.2	.60	23957	36215	.711	.709	.272
.950	3.4	.60	25646	39912	.711	.709	.272
.950	3.6	.60	27357	41677	.711	.709	.272
.950	3.8	.60	46527	56455	.742	.92	.644
.950	4.0	.60	34976	51759	.733	.742	.712
.950	4.2	.60	38198	54981	.733	.752	.716
.950	4.4	.60	36549	46199	.733	.744	.716
.950	4.6	.60	33200	56956	.727	.744	.713
.950	4.8	.60	36117	53767	.727	.744	.713
.950	5.0	.60	38963	56613	.727	.744	.713
.950	5.2	.60	26192	45477	.727	.744	.713
.950	5.4	.60	29346	47860	.722	.744	.713
.950	5.6	.60	31798	58292	.722	.744	.713
.950	5.8	.60	34246	52769	.722	.744	.713
.950	6.0	.60	36729	55245	.722	.744	.713
.950	6.2	.60	24811	43391	.722	.744	.713
.950	6.4	.60	24698	45479	.717	.744	.713
.950	6.6	.60	26213	47523	.717	.745	.713
.950	6.8	.60	36371	49751	.717	.745	.713
.950	7.0	.60	32558	51916	.717	.745	.713
.950	7.2	.60	21593	41798	.715	.745	.713
.950	7.4	.60	23368	43625	.715	.745	.713
.950	7.6	.60	25240	45493	.715	.745	.713
.950	7.8	.60	27149	47394	.715	.745	.713
.950	8.0	.60	29878	49319	.715	.745	.713
.950	8.2	.60	19483	48593	.713	.745	.713
.950	8.4	.60	21699	42210	.713	.745	.713
.950	8.6	.60	22753	43864	.713	.745	.713
.950	8.8	.60	24446	45556	.713	.745	.713
.950	9.0	.60	17721	39697	.711	.745	.713
.950	9.2	.60	17462	41137	.63795	.63795	.713
.950	9.4	.60	19161	41654	.711	.745	.713
.950	9.6	.60	16948	29944	.65266	.711	.745
.950	9.8	.60	26666	22753	.61488	.61488	.711
.950	10.0	.60	26453	24242	.66766	.66766	.711
.950	10.2	.60	21944	32987	.68297	.68297	.711
.950	10.4	.60	21533	36529	.69887	.69887	.711
.950	10.6	.60	29695	36891	.71488	.71488	.711
.950	10.8	.60	26666	37661	.72979	.72979	.711
.950	11.0	.60	26242	39236	.74557	.74557	.711
.950	11.2	.60	16984	27384	.64887	.64887	.711
.950	11.4	.60	17317	30697	.65794	.65794	.711
.950	11.6	.60	18664	36096	.67144	.67144	.711
.950	11.8	.60	26042	31422	.68519	.68519	.711
.950	12.0	.60	21435	32815	.69917	.69917	.711
.950	12.2	.60	22649	34221	.71317	.71317	.711
.950	12.4	.60	24255	35635	.72732	.72732	.711
.950	12.6	.60	25675	37655	.74153	.74153	.711

## **APPENDIX B**

In this appendix the effect of various combinations of mount error on the double arch mirror surface mounted on the proposed flexures is studied.

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१. ग्रन्ति	२. वर्षांस्त्र	३. ग्रन्ति	४. १८६०	५. ग्रन्ति	६. ३२४६	७. ग्रन्ति	८. ४६२५
१. ४६६९	२. १८८५	३. २१४०	४. १०००	५. ४८२६	६. ३२४७	७. १९७९	८. १७५०
१. ४०८८	२. ११३७	३. ११९८	४. ११९८	५. ४८८८	६. ३१९९	७. ११११	८. १०११
१. २७६२	२. १८९०	३. ११२५	४. १११०	५. ४८८७	६. ३१८६	७. ११६६	८. १००६
१. २५१०	२. १८८७	३. ११३५	४. १४७२	५. ४८८८	६. ३१८७	७. ११६७	८. १००७

## RESIDUAL WAVEFRONT VARIATIONS OVER UNIFORM MESH

PT3 RMS MAX MIN SPAN VOLUME  
664. .784 1.642 -2.694 4.336 .. 7.833

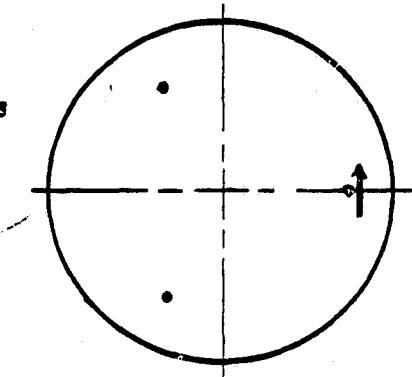


Figure A1. Cool down and radial tilt in one flexure location.  
 Units =  $10^6$  in.

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## ZERNIKE POLYNOMIAL COEFFICIENTS

১. ৩০০০	২. ৪৫০০	৩. ৮০০০	৪. ২০০০	৫. ৭২৪৭	৬. ৮০০০	৭. ৮০৭৯	৮. ৯৬৬২
- .1147	- .7492	- .8000	- .3634	- .2800	- .9949	- .9216	- .9959
- .2500	- .9029	- .1873	- .9999	- .1739	- .9999	- .9926	- .1112
- .3000	- .1775	- .8000	- .9794	- .8898	- .9728	- .8898	- .8546
- .6000	- .9818	- .9838	- .9969				

## RESIDUAL WAVEFRONT VARIATIONS OVER UNIFORM MESH

PTG RMS MAX MIN SPAN VOLUME

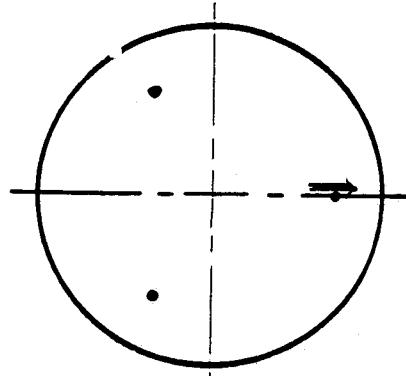


Figure A2. Cool down and tangential tilt in one flexure location.  
 Units =  $10^6$  in.

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**ZERNIKE POLYNOMIAL COEFFICIENTS**

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-0.1870	0.0000	0.0211	0.0000	0.2595	0.0000	0.1351	0.2176
0.0000	0.0036	0.0000	0.0217	0.0000	0.0305	0.0000	0.2291
0.0005	0.0000	0.0003	-0.0000	0.0160	0.0000	0.0117	0.0000
0.0245	0.0000	0.0057	0.0100				

**RESIDUAL WAVEFRONT VARIATIONS OVER UNIFORM MESH**

PTS	RMS	MAX	MIN	SPAN	VOLUM
604	.195	.326	.364	.683	.927

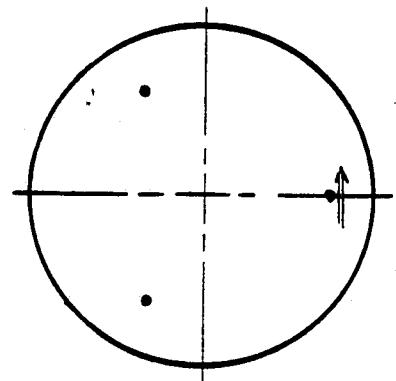


Figure A3. Cool down and flexure error in one flexure location.  
Units =  $10^6$  in.

## ZERNIKE POLYNOMIAL COEFFICIENTS

## RESIDUAL WAVEFRONT VARIATIONS OVER UNIFORM MESH

PT3	RH3	MAX	M2N	SPAN	VOLUM
066.	073	2.794	02.047	4.000	0.040

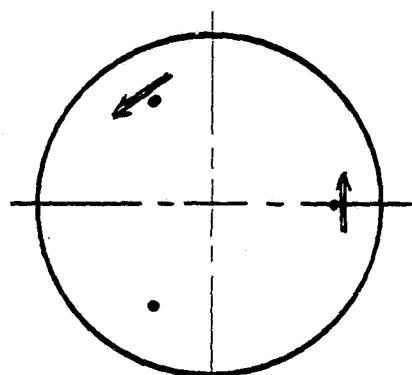


Figure A4. Cool down and radial tilt in two flexure locations.  
 Units =  $10^6$  in.

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## ZERONINE POLYNOMIAL COEFFICIENTS

7,7023	8,5595	8,8595	8,9917	2,3623	7,9649	7,8948	7,8642
-1,1147	1,4984	1,4883	1,2819	1,0836	1,0821	1,0216	-1,2234
,1290	.2825	.3746	.1596	.2889	.0822	.0813	.1112
.1537	.2887	.2688	.2397	.0898	.1440	.0873	.2273
-1,0816	.2229	-1,0838	.0869				

## RESIDUAL WAVEFRONT VARIATIONS OVER UNIFORM MESH

PTS	RNG	MAX	MTH	SPAN	VOLUME
664.	2.873	5.358	45.250	18.498	14.994

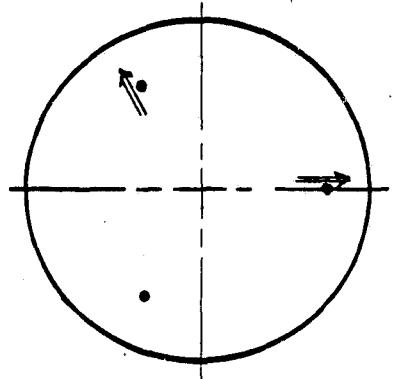


Figure A5. Cool down and tangential tilt in two flexure locations.  
 Units =  $10^6$  in.

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 .100    .900    2.000    -.450    -.100    .150    .450  
 .000    .000    .000    .000    .000    .000    .000

KJ I H GFF EE    EE  
 LKJ TH G FF EEEEEEEEEE FF G  
 OBNHLK JT H G FF FFF G  
 UTSRPNH KJ I H G FFF FFFF GG HH II JJ  
 UTSRNP LK J HH GG FFFFF GG HH II JJ  
 TS NML KJ I HH GGG GGG HH II JJ  
 12    BN LK J I H GGRGR HH II JJ KKKK  
 32    GP M K J II HHHH HHHH II J HH LL  
 380 N L K J I III JJ KK LLLL LL  
 9432182XWV TS P NML K JJJJJJJJJ JJ KK LL MHHHHHHHH  
 W TS RQ BN MLL KKK KKK LL MHH MHHHHHHHHHH  
 43210ZV V TS Q BN MLL LL HH MHH NNNN P NNNN  
 S T 92X44U3RQ P NN MHHHHHHHH NN PP OO PRRPPP N  
 43210Z W VU T GR Q P NN RR RRRR OG PPP +  
 321A XW V T S RQ P NN  
 YW V UT S RQ P NN  
 XH VU T S RQ P NN  
 YH U T S RQ P NN  
 21 YXWV UT S RQ P NN M  
 ZYXWV T S Q Q P N MHH  
 YXWV S R Q P V M  
 XH V SP Q P N M LLL  
 WY SR Q P N MLL KKK  
 V QD P N MLL KK K TUU UUUU  
 U SPQ N M L KK JJJJJJJ KK L HH P QR S T UUUU  
 SPQ N M L KK JJ TJKLHN QR SS TT UUUU  
 P N L K X J J TTTTTT TJKLHN P QR SS TT UUUU  
 PN L K J II HHMHHHHHHHH II JK L MN P QR S T UUU  
 MI K J I HMM HH II JK L MN P QR S T UUU VVV  
 I X II H GGGGGGGG HH II JK L MN P QR S T UUU VVV  
 I JTT H GGG GGG HH II JK L MN P QR S T UUU VVV  
 I H GG FFFFFF GG HH II JK L MN P QR S T UUU VVV  
 H G FFFF FFFF GG HH II JK L MN P QR S T UUU V  
 G FF FFF G HH II JK L MN P QR S T UUU V  
 E EEEEEEE FF GG H II JK L MN P QR S T UUU  
 E EEEEEEE F G H II JK L MN P QR S T UUU  
 E EEE F G H II JK L MN P QR S T UUU

#### ZERNIKE POLYNOMIAL COEFFICIENTS

.00000	.00000	.00000	.29857	-2.3623	-1.3178	.6948	.4624
-.5069	.7692	.7023	.2819	.5998	.9821	.1579	.4014
-.1230	.9137	.1873	.3793	.8869	.3866	.2013	.1211
.2975	.9897	.8660	.0397	.9729	.9729	.1659	.0273
.2502	.9209	.9235	.9472				

#### RESIDUAL WAVEFRONT VARIATIONS OVER UNIFORM MESH

PTS	RMS	MAX	MIN	SPAN	VOLUME
664.	1.748	5.210	43.125	8.335	9.222

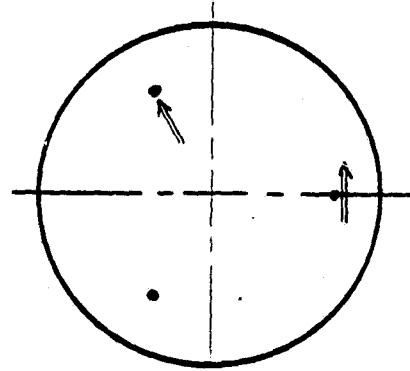


Figure A6. Cool down and radial and tangential tilt in two flexure locations. Units =  $10^6$  in.

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## ZERNIKE POLYNOMIAL COEFFICIENTS

9.9999	9.9999	9.9999	3.5612	3.3899	3.5697	3.1911	.9848
-1.6543	1.4984	1.5840	.9966	.2659	.9239	.3885	.39730
-0.2252	.3268	.3746	.2889	.1034	.1359	.2688	.1974
.1117	.1729	.1675	.1621	.1425	.1448	.1403	.0774
.1110	.2189	.2468	.2922				

## RESIDUAL WAVEFRONT VARIATIONS OVER UNIFORM MESH

PTS RMG MAX MIN SPAN VOLUME  
664. 2.324 5.988 .46.336 12.316 19.698

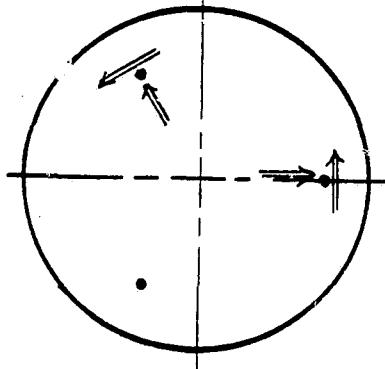


Figure A7. Cool down and radial and tangential tilt in two flexure locations. Units =  $10^6$  in.

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## ZERONIKE POLYNOMIAL COEFFICIENTS

## RESIDUAL WAVEFRONT VARIATIONS OVER UNIFORM MESH

PTS RMS MAX MIN SPAN VOLUME  
664. .844 1.965 -1.938 3.883 5.424

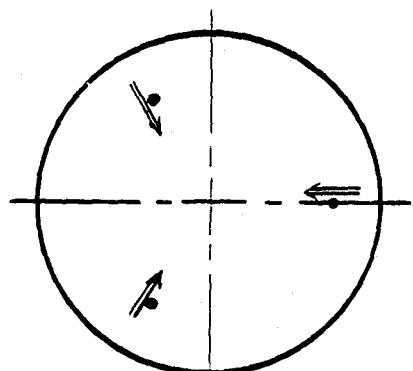


Figure A8. Cool down and tangential tilt in all three flexure locations.  
 Units =  $10^6$  in.

ORIGINAL PAGE IS  
OF POOR QUALITY

CONTOUR STEP	WIDTH	PAGE SIZE	-M-	-N-	-P-	-Q-
300	.500	2,000	.450	.150	.150	.450
*	*	*	*	*	*	*
	J	GG GGG H I J K MN PQR				
*		H GGG GGG MM I J K L MN PQ RS W				
*		H MM I J K L N PQ S U				
*	L T HHHHHHH IT J K H N PQ RS TU V XYZ					
*	L J IIIIIIIII JJ K L H N PQ S T UV W Y YZ					
*	L K JJJJ JJJ K K L L M N P R S T U V W X Y					
*	PN M L KKK KKK L H N PQ S T U V W X Y					
*	RQ P N M LLL LLL MM NN PP QO R SS T U V W W W X					
*	R Q P MN HHHHHHHHH NNNNN PPP QQ RR SS TT UU VVVV					
*	UTS R Q P NNNNNNNNNNN PPP QQO RRR SS TT UUU V					
*	UT R Q Q PPP PPPP QQQ RRRR SSS TTT					
*	UT 4 R Q PPPPPP QQQ RRR SSSSSSSSS TTT SSS					
*	T S R Q Q QQQ RR SS TTTTTTTT SSSSSSS RRR R					
*	T S R Q Q QQQ RR SS TTT UUU TTTT SSS RRR QO					
*	VU TS R Q QQQ RR UU TT SS RRR QO PP NNN					
*	UT S R Q PPPPPP QO R UU TT SS R QO PP N MM					
*	TS H Q PPPPPP QO Q UU TT S R Q P NN MM LL					
*	UT R Q P NNNNNN PP Q TT S R Q P N MLL K J					
*	SR Q P NN HN HN P TT S R Q P N MLL K J I					
*	TS Q P N MMHHHHHHHHH N SR Q P N MLL K J I HH I					
*	PQ P N MM LLLL MM N SR Q P N MLL K J I HH G					
*	SP P MM LL LL H R Q P N MLL K J I HH G					
*	SP NM L KKKKK KKKK L Q P N MLL K J I HH G FF					
*	RD N MLL KK JJJ KK NH 4 L K J I HH G FF E					
*	P N M L K JJJ JJJ KK L M NNNNNNN MM L K J I HH G FF EEE					
*	P VM L K JJJ JJJ KK L MMN 4 H H L L K J J T H G FF EEE					
*	HM L K J JJJ JJJ KK LL HHHHHHHHH L K J I HH G FF					
*	4 L K J JJJ JJJ KK LL HHHHHHHHH L K J I HH G FF					
*	N H L K JJJ JJJ JJJ KK LLL HHHHHH L K J I HH GGG					
*	H L KK KKK LLL MMN H MM LL KK J J I HH					
*	MM LL KHHHHHH LLL MM NNNN NNNN MM L KK JJJ II					
*	N MM LLL LLLL MMN NN PPPPPP PPPP NN MM LL KK JJ					
*	N MMN MMNN NN PPP QQQQQQQQ PPP NNN MMN LL					
*	NNNN NNHN PP QO RRR QQQ PPP NN					
*	PPPPPPP QO RR SSSSSSS RRRR QO P					
*	R RRR SSS TTTTTTTTTTTTTT SSS R					
*	SSSSSS S TT UUUU UUUU					
*	UUUU VVV WWWWWWWWWWWWW					
*	WWWWWW XXXX YYYYYYYY					
*	Z					
*	+	+	+	+	+	+

ZERNIKE POLYNOMIAL COEFFICIENTS

.00000	.00000	.00000	.00018	.9730	.0274	.0007	.0705
-1.4178	2.2476	-0.1014	.1754	.2054	.4943	.3012	.2843
-0.1460	.0255	-.5620	.1041	.1803	.1463	.2534	.1957
0.0454	.0789	-.2013	-.0023	.1394	.2160	.0562	.2973
.1192	.2065	-.0043	.0095				

RESIDUAL WAVEFRONT VARIATIONS OVER UNIFORM MESH

PTS	RMS	MAX	MIN	SPAN	VOLUME
664.	1.268	3.875	-2.095	6.770	8.543

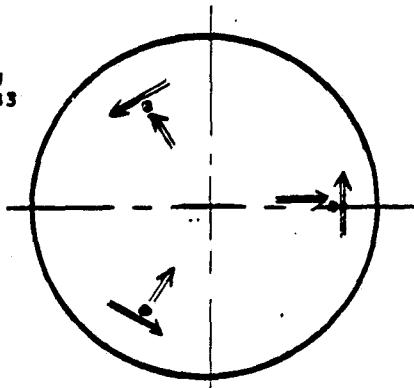


Figure A9. Cool down and radial and tangential tilt in all three locations. Units =  $10^6$  in.